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Demographic change, government debt and fiscal sustainability in Japan: The impact of bond purchases by the Bank of Japan ☆

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A B S T R A C T

We reconsider the fiscal consequences of an aging population in Japan that were reported in Hansen and İmrohoroğlu (2016). That paper predicted that the net debt to GNP ratio would reach 250 percent in 2021, while the actual net debt to GNP ratio was roughly constant at about 120% from 2011 to 2019. Here we study the role played by higher tax revenues, lower spending, and lower interest rates than were assumed in the previous paper. Most importantly, we consider the role played by Japanese government bonds held by the Bank of Japan in enabling this period of debt stability. We conclude that the stable net debt to output ratio is predicted to be temporary and reach 250 percent before 2040.

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1. Introduction

Hansen and İmrohoroğlu (2016) developed a neoclassical general equilibrium growth model with endogenous pricing of Japanese government bonds (JGBs) to evaluate the quantitative implications of aging in Japan. The model was calibrated to data from 1981 to 2010, and projections of future government expenditures from 2011 to 2050 attributable to an aging population were taken from Fukawa and Sato (2009). The main finding of that paper is that without increased revenue from higher taxes or a broadening of the tax base, these projected expenditures would lead to Japanese debt relative to output to exceed 250 percent in 2021. Achieving fiscal sustainability was found to require a large increase in tax revenues, in the range of 30 to 40 percent of aggregate consumption.

Recent Japanese data since that paper was published seems to imply that these conclusions were much too pessimistic. Fig. 1 shows the net debt-to-output ratio from 1981-2019 in Japan and the projections from Hansen and İmrohoroğlu (2016). Instead of soaring towards 250 percent in the decade of the 2010s, net Japanese government debt to GNP stabilized
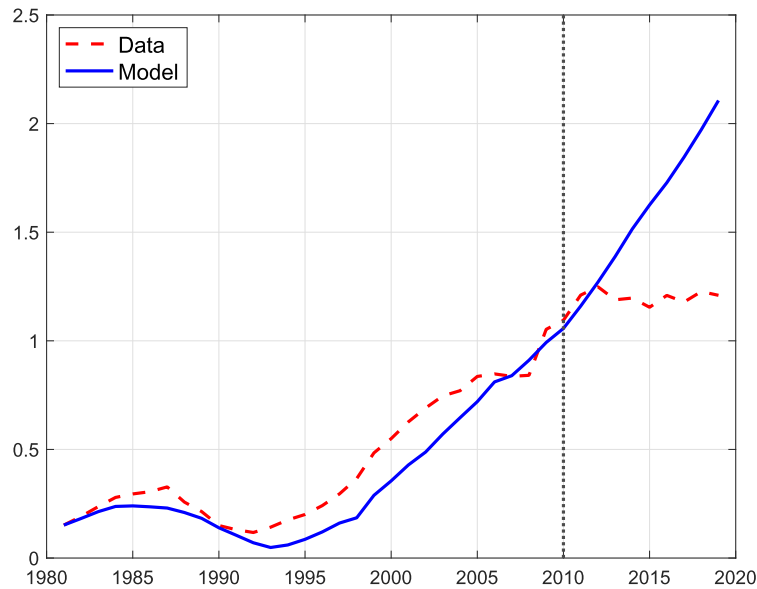
Fig. 1. Net debt to GNP ratio.

at about 120 percent from 2011 - 2019.[1] The model does a good job of tracking the path of the debt-to-GNP ratio until 2010, indicated with a vertical dotted line. After that date, the model shows a rapidly rising debt-to-GNP ratio, but the observed path after 2010 has essentially been flat.

In this paper, we use our neoclassical growth model along with more recent data to understand the reasons for the Japanese success in stabilizing government debt during this period. This will include comparing our projections of exogenous and endogenous variables that determine the net debt to output ratio in Japan. In particular, we use realized values of total factor productivity (TFP), government consumption and transfer payments, and endogenous predictions of interest payments on government debt during this period. In addition, we use data on tax revenues from labor and corporate income taxes to compute labor and capital income tax rates from 1981 to 2019.[2]

The most striking change since 2010, however, is the huge growth in the percentage of Japanese net government debt held by the BOJ. Prior to 2010, that percentage was never larger than 35.2 percent, which occurred in 1990. By 2020, that percentage was almost 70 percent (see Fig. 2). This is potentially important since interest payments made to the BOJ are returned to the central government.

We modify the model of Hansen and İmrohoroğlu (2016) to incorporate BOJ holdings of Japanese government debt and use recent data to quantify the role played by government spending reductions, realized TFP, revenue increases, and lower interest payments on government debt in accounting for the flattening of Japan's net debt-to-output ratio. Also, we ask if this ratio will continue to be flat in the future. We find that lower interest rates (higher bond prices) than we anticipated and the huge increase in BOJ purchases of debt were the primary enablers of fiscal stabilization in recent years. In addition, given the spending forecasts of Fukawa and Sato (2009), net debt to output is predicted to resume rising unless additional revenue can be raised.

There is a large literature studying the additional revenue required to stabilize the debt to output ratio in Japan. As previously noted, our approach, which follows İmrohoroğlu and Sudo (2011) and Hansen and İmrohoroğlu (2016), uses forecasts of aging related government spending increases (mostly pension and health care spending) from Fukawa and Sato (2009). These forecasts imply an increase in the government spending to output ratio due to aging of 7 percentage points from 2010 to 2050. Another group of papers uses institutional details of Japanese pension and health care programs in the context of a life cycle model to endogenously compute the fiscal costs associated with an aging Japan. İmrohoroğlu et al. (2016) find that spending increases due to aging are similar to Fukawa and Sato (2009). Braun and Joines (2015) and Kitao (2015) also follow this approach and obtain similar results concerning tax increases required to achieve fiscal sustainability.

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[1] We use data from the 1993 SNA so that our results are directly comparable with the original paper. Available data stops at 2020, but we elected to consider only data to 2019 to avoid COVID considerations. The new 2008 SNA data treats R&D expenditures as produced assets and hence their inclusion in private investment. For our purposes, there is very little difference in our results from using the old versus the new SNA data.

[2] In our previous work, Hansen and İmrohoroğlu (2016), we used tax rates on capital income measured as in Hayashi and Prescott (2002) and labor income tax rates from Gunji and Miyazaki (2011).
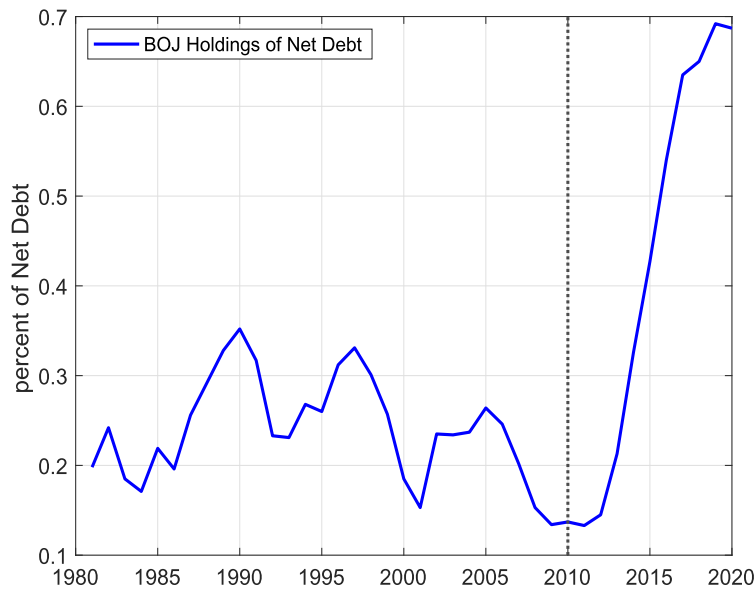
Fig. 2. BOJ holdings of JGBs.

A third set of papers uses estimates based on those in Broda and Weinstein (2005) which are quite a bit more optimistic than those of Fukawa and Sato (2009). Contributions here include Doi (2008), Doi et al. (2011), and Bamba and Weinstein (2021). These authors conclude that current tax rates are consistent with fiscal sustainability in the long run.[3]

The remainder of this paper is organized as follows. Section 2 discusses how our assumptions on government consumption and transfer payments, as well as total factor productivity from 2011 to 2019 differ from those made in our previous paper. These are all exogenous time series in our analysis so that they can be studied without details of the model being described. Next, the model is described in section 3, and section 4 describes the data sources and calibration of the model. The time path of endogenous components of the government budget constraint—tax revenue and interest payments on debt in particular—are the focus of section 5. Our main results are presented in section 6. Here we conduct a series of experiments to quantify the role played by various factors in stabilizing Japanese government debt in recent years. Concluding comments follow in Section 7.

2. Government expenditures and total factor productivity

The model simulations are driven in part by the exogenously fed government spending and total factor productivity series. Here we document how the assumptions made by Hansen and İmrohoroğlu (2016) compare with realizations from 2011 to 2019.

Total factor productivity Assuming Cobb-Douglas production, total factor productivity (TFP) is computed using data on capital, labor, and output and the value for capital's share, θ, from the calibrated model in Hansen and İmrohoroğlu (2016).[4] That is, $TFP_t = Y_t/(K_t^\theta h_t^{1-\theta})$, where $\theta = 0.3783$. Since the time series studied in that paper ended in 2010, we assumed that TFP grows at a constant rate beyond that point. In Fig. 3, we show TFP from our original data period in the solid blue curve, as well as how that changes if we include data up to 2019, in the dashed red curve. We find that TFP from 2011 to 2019 is lower than what we forecasted, so better than expected TFP growth cannot explain Japan's success in stabilizing its debt.

Government purchases and transfer payments Government purchases and transfer payments to GNP ratios are depicted in Fig. 4, with the earlier data from 1981-2010 and more recent data from 1981-2019 represented by the blue solid curve and the red dashed curve, respectively. We see that government purchases were higher in 2011 to 2019 than what was assumed in our previous paper. Transfer payments, however, were significantly lower.[5]

Fig. 5 shows total government spending (purchases plus transfers) to GNP. The Japanese government did spend less than we predicted from 2012 to 2018. However, by 2019 spending is equal to what was predicted for that year. Hence, Japanese success in economizing on spending surely helped to reduce government debt relative to what we had predicted during this period.

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[3] These papers employ an accounting framework suggested by Blanchard (1990) to calculate a constant tax rate that achieves fiscal sustainability. Bamba and Weinstein (2021) use the same methodology replacing the earlier projections for 2001-2015 with actual government expenditures and revenues and
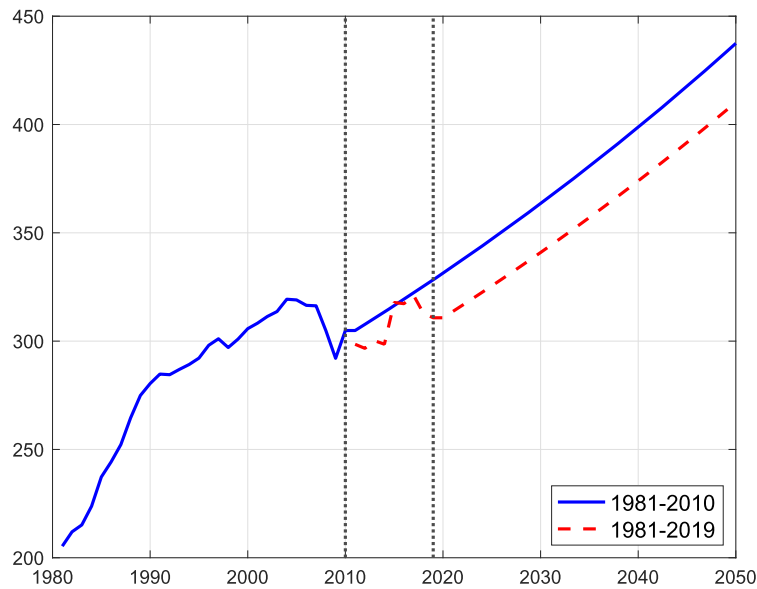
Fig. 3. Total factor productivity. (For interpretation of the colors in the figure(s), the reader is referred to the web version of this article.)
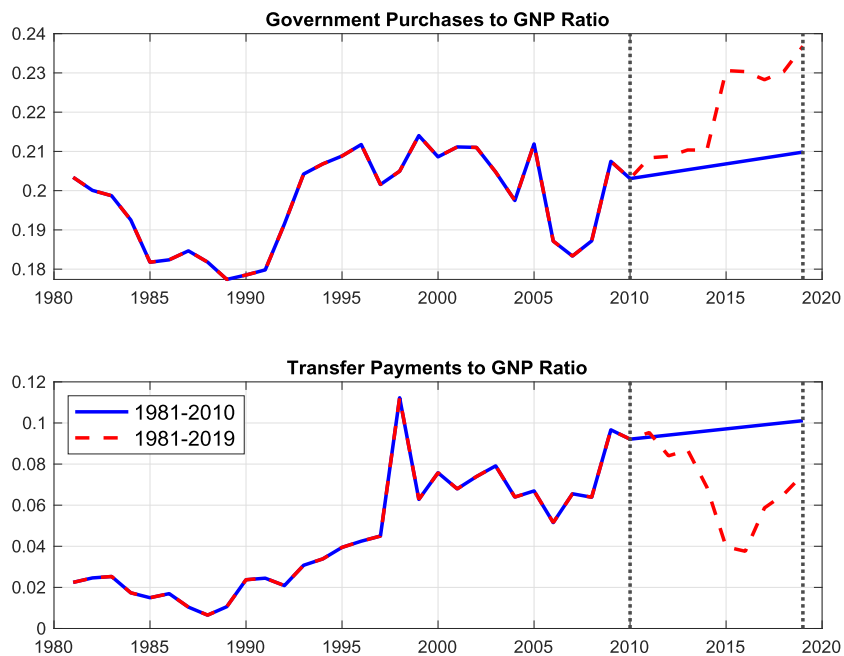

Fig. 4. Government purchases and transfer payments (fraction of GNP).

3. Model

In this section, we describe the details of the one sector neoclassical growth model from Hansen and İmrohoroğlu (2016) augmented to allow for bond purchases by the Bank of Japan. We do not model monetary policy or the central bank's

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find that the updated estimates of their sustainable tax rate is *lower* than the earlier estimates. This is due to increases in tax revenues generated by the 2004 pension reform and a higher consumption tax rate as well as contained government expenditures, especially on pensions.

⁴ Our measures of Japanese capital, labor and output are detailed in section 4 and in the Appendix.

⁵ Note that net indirect taxes are subtracted from transfer payments in Fig. 4. Hence, the lower transfer payments to GNP ratio in that figure may be due a combination of lower transfer payments and higher indirect taxes.
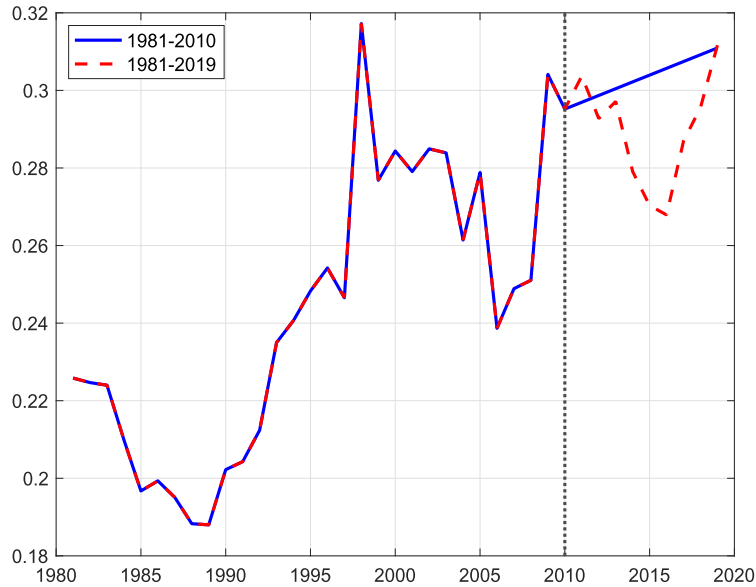
Fig. 5. Government purchases plus transfer payments (fraction of GNP).

balance sheet in detail. Instead, we introduce an agent that purchases some exogenously determined amount of Japanese government bonds and rebates any interest payments it receives back to the central government. The potential inflationary consequences of such a policy are not modeled. In fact, nominal interest rates have remained quite low during the period we study.

The notation we employ uses uppercase variables to denote per capita values that grow along a balanced growth path, and lowercase variables are stationary. The time period of the model is one year.

The economy is populated by a representative household with N_t members at time t. The size of the household is assumed to grow at a time-varying growth factor η_t so that $N_{t+1} = \eta_t N_t$. There is no uncertainty in our economy; households are assumed to have perfect foresight.

The analysis in this paper takes as given time series on tax rates $(\tau_{c,t}, \tau_{h,t}, \tau_{k,t}, \tau_{b,t})$, government consumption (G_t), transfer payments (TR_t), the fraction of government debt held by the Bank of Japan (λ_t), the working age population (N_t), and total factor productivity (A_t), where actual time series are used from 1981-2019. Forecasts and assumptions extend these series to 2050 and beyond. The tax rates, the ratios of government purchases and transfer payments to output $(G_t/Y_t, TR_t/Y_t)$, the fraction of debt held by the central bank, the growth rates of N_t and A_t are all eventually constant so that the economy converges to a balanced growth path. Hours worked (h_t), consumption (C_t), output (Y_t), the stock of capital (K_{t+1}), tax revenues, government debt (B_t), and the price of government bonds (q_t) from 1981 into the infinite future are endogenously determined by the model.

3.1. Government

The government issues one-period discounted bonds in period t equal to B_{t+1}, where $B_{t+1} = B_{t+1}^h + B_{t+1}^c$. B^h is bonds held by households and B^c is bonds held by the central bank. The government is assumed to collect revenue from taxing household consumption at the rate $\tau_{c,t}$, labor income at the rate $\tau_{h,t}$, capital income at the rate $\tau_{k,t}$, and interest on government bonds at the rate $\tau_{b,t}$. In addition, given the fraction of government debt held by the Bank of Japan (λ_t), we have $B_t^h = (1 - \lambda_t)B_t$ and $B_t^c = \lambda_t B_t$.

Given time series for G_t and TR_t, B_{t+1} is determined by the following budget constraint (where all quantities are in per capita terms):

$$G_t + TR_t^* + B_t = \eta_t q_t B_{t+1} + \tau_{c,t} C_t + \tau_{h,t} W_t h_t \tag{1}$$
$$+ \tau_{k,t}(r_t - \delta)K_t + (\tau_{b,t}(1 - \lambda_t) + \lambda_t)(1 - q_{t-1})B_t.$$

In addition to variables already defined, W_t and r_t denote the wage rate and the return to capital, and δ is the depreciation rate of capital. Note that the last term of this equation is interest payments returned to the government. In particular $\tau_{b,t}(1 - \lambda_t)(1 - q_{t-1})B_t$ is interest payments to private agents collected as taxes by the government, and $\lambda_t(1 - q_{t-1})B_t$ is interest payments to the Bank of Japan rebated back to the government.

The government is also assumed to be subject to a "debt sustainability" rule that forces the government to retire debt when the debt-to-output ratio reaches some arbitrary value $b_{\max}$ that we specify. We include this feature for two reasons.

First, our solution procedure for computing equilibrium paths requires that the economy ultimately converges to a steady state with a constant bond-to-output ratio. Without some additional constraint, this convergence would not be guaranteed. Second, while there is no natural limit to how much debt the government in our model can issue, such a limit almost certainly exists in actual economies. As in the benchmark experiment in Hansen and İmrohoroğlu (2016), we assume that debt reductions required to stabilize the debt-to-output ratio are financed by reducing transfer payments. This is equivalent to a lump-sum tax in our model. Hence, if the government is required by our debt sustainability rule to spend D_t on debt reduction, TR_t^* in the government budget constraint (1) is as follows:

$$TR_t^* = TR_t - D_t. \tag{2}$$

The debt sustainability rule works in the following way. The value of D_t is initially zero and remains so until the debt-to-output ratio reaches the value $b_{\max}$. At this point, the government is forced to begin retiring a sufficient amount of debt by reducing transfers (setting $D_t > 0$) so that the debt-to-output ratio falls and ultimately converges to some value $\bar{b} = \overline{B}_t/\overline{Y}_t$. More formally, D_t is set as follows:

$$D_t = \begin{cases} \kappa(B_t - \overline{B}_t) & \text{if } B_s/Y_s \geq b_{\max} \text{ for some } s \leq t, \\ 0 & \text{otherwise.} \end{cases} \tag{3}$$

In practice, we choose the parameter $\kappa > 0$ to be as small as possible so that $B(t)/Y(t)$ converges to $\bar{b}$ and does not continue to increase without limit.

3.2. Household's problem

The household is endowed at time 0 with initial holdings of per capita physical capital $K_0 > 0$, and real, one-period, zero-coupon, discount bonds $B_0^h > 0$. In addition, each member of the household is endowed each period with one unit of time that can be allocated for market activities h_t or leisure $1 - h_t$. Given a sequence of wages, rental rates for capital, government bond prices $\{W_t, r_t, q_t\}_{t=0}^{\infty}$, tax rates on consumption, and labor, capital and bond income, and per-capita transfer payments $\{\tau_{c,t}, \tau_{h,t}, \tau_{k,t}, \tau_{b,t}, TR_t\}_{t=0}^{\infty}$, the household chooses a sequence of per member consumption, hours worked, capital, and real bond holdings $\{C_t, h_t, K_{t+1}, B_{t+1}^h\}_{t=0}^{\infty}$ to solve the following problem:

$$\max \sum_{t=0}^{\infty} \beta^t N_t \left[\log C_t - \alpha \frac{h_t^{1+1/\psi}}{1 + 1/\psi} + \phi \log(\mu_t + B_{t+1}^h) \right] \tag{4}$$

subject to

$$(1 + \tau_{c,t})C_t + \eta_t K_{t+1} + q_t \eta_t B_{t+1}^h = (1 - \tau_{h,t})W_t h_t + \left[(1 + (1 - \tau_{k,t})(r_t - \delta) \right] K_t$$
$$+ [1 - (1 - q_{t-1})\tau_{b,t}] B_t^h + TR_t^*,$$

where $K_0 > 0$ and $B_0^h > 0$ are given initial conditions. Here K_{t+1} is per member holdings of capital at time $t + 1$. $\eta_t K_{t+1}$ expresses the same quantity of capital per member at time t. The household's maximization is subject to a budget constraint where after-tax consumption expenditures and resources allocated to wealth accumulation in the form of capital and bond holdings are financed by after-tax labor income, after-tax capital income and holdings of capital, after-tax proceeds of bond holdings chosen in the previous period, and transfer payments from the government. The parameter β denotes the household's subjective discount factor. The disutility of work is described by $-\alpha < 0$ and $\phi > 0$ denoting the household's preferences for government bonds. We use ψ to denote the intertemporal elasticity of substitution (IES) of labor.

Since about 90% of the Japanese government bonds are held domestically, we assume that Japan is a closed economy. In addition, Japanese government bonds historically have had yields less than the return to physical capital. As a result, we introduce government debt in the utility function with $\phi > 0$.[6]

Finally, μ_t is a parameter that limits the curvature of the period utility function over bonds. Essentially, it represents assets that might be perfect substitutes for Japanese government-issued bonds in generating utility for households.[7] We allow this parameter to move at the same rate of balanced growth as the rest of the economy so that the detrended version is a constant. In particular, $\mu_t = \mu A_t^{1/(1-\theta)}$.

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[6] This formulation captures the convenience yield provided by government bonds as introduced by Krishnamurthy and Vissing-Jorgensen (2012).

[7] This parameter helps to more closely match the volatility of bond prices.

3.3. Firm's problem

A stand-in firm operates a constant returns to scale Cobb-Douglas production technology

$$N_t Y_t = A_t (N_t K_t)^\theta (N_t h_t)^{1-\theta}$$

$$N_{t+1} K_{t+1} = (1-\delta) N_t K_t + N_t X_t.$$

Capital depreciates at the rate δ. The income share of capital is given by θ. A_t is total factor productivity which grows exogenously at the rate γ_t, so we have $A_{t+1} = \gamma_t A_t$. Per capita gross investment is denoted by X_t.

3.4. Equilibrium

Given a government fiscal policy $\{G_t, TR_t, D_t, B_t, \tau_{h,t}, \tau_{k,t}, \tau_{c,t}, \tau_{b,t}\}_{t=0}^\infty$, a monetary policy $\{\lambda_t\}_{t=0}^\infty$, a debt sustainability rule $\{\kappa, \bar{b}, b_{\max}\}$, and the paths of working age population $\{N_t\}_{t=0}^\infty$ and technology $\{A_t\}_{t=0}^\infty$, a competitive equilibrium consists of an allocation $\{C_t, h_t, K_{t+1}, B_{t+1}\}_{t=0}^\infty$, factor prices $\{W_t, r_t\}_{t=0}^\infty$ and the bond price $\{q_t\}_{t=0}^\infty$ such that

- the allocation solves the household's problem,
- the allocation solves the firm's profit maximization problem with factor prices given by: $W_t = (1-\theta) A_t K_t^\theta h_t^{-\theta}$, and $r_t = \theta A_t K_t^{\theta-1} h_t^{1-\theta}$,
- the government budget is satisfied,
- given a value for $b_{\max}$, the value of κ in the fiscal sustainability rule is sufficiently large to guarantee convergence of B_t/Y_t to $\bar{b}$,
- the market for bonds clears, $(1 - \lambda_{t+1}) B_{t+1} = B_{t+1}^h$
- and the goods market clears: $C_t + [\eta_t K_{t+1} - (1-\delta) K_t] + G_t = Y_t$.

3.5. Solution procedure

A set of detrended equilibrium conditions is provided in the appendix. In particular, a change of variables transforms each trending per capita variable Z_t to obtain its detrended per capita counterpart:

$$z_t = \frac{Z_t}{A_t^{1/(1-\theta)}}.$$

A value for k_{1981} and a sequence $\{\tau_{c,t}, \tau_{h,t}, \tau_{b,t}, \tau_{k,t}, \eta_t, \gamma_t, g_t, tr_t, \lambda_t\}_{t=1981}^\infty$ are taken as given, where the elements of this sequence are constant beyond some date. These constant values determine the steady state to which the economy ultimately converges. Next, a shooting algorithm, similar to that in Hayashi and Prescott (2002) and Chen et al. (2006), determines the value of k_{1982} such that the sequence of endogenous variables converges to the steady state. This guarantees that the sequence of capital stocks satisfies the transversality condition. Note that the fiscal sustainability rule guarantees that the bond-to-output ratio is equal to $\bar{b}$ in the steady state achieved in the limit.

4. Data and calibration

Calibration of the model follows the methodology articulated in Cooley and Prescott (1995) and more directly Hayashi and Prescott (2002). For the most part, the data and calibration are the same as in Hansen and İmrohoroğlu (2016) and further details can be found there as well as in the appendix of this paper. Parameter values are obtained using annual data from 1981 to 2010 corresponding to all exogenous and endogenous variables. In this section, we review some key aspects of the data necessary for interpreting the results presented in this paper and explain how the calibration differs from Hansen and İmrohoroğlu (2016).

Data: The national income accounting used to construct Japanese time series for Y, C, K, and G is the same as that developed by Hayashi and Prescott (2002). In particular Y is Gross National Product computed as the sum of private consumption expenditures (C), private gross investment that includes net exports with net factor payments from abroad, and government expenditures on final goods and services (including land purchases).

Population, N_t, is working age population between the ages of 20 and 69. We use the actual values between 1981 and 2019 to compute a series for η_t and rely on official projections for 2020-2050. We assume that the population stabilizes after 2050; that is $\eta_t = 1$, $t \geq 2050$.[8]

For h_t we take the product of employment per working age population and average weekly hours worked, normalized by dividing by 98, which is our assumed value for discretionary hours available per week.

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[8] Population projections are the medium-fertility and medium-mortality variants of forecasts published by the National Institute of Population and Social Security Research.
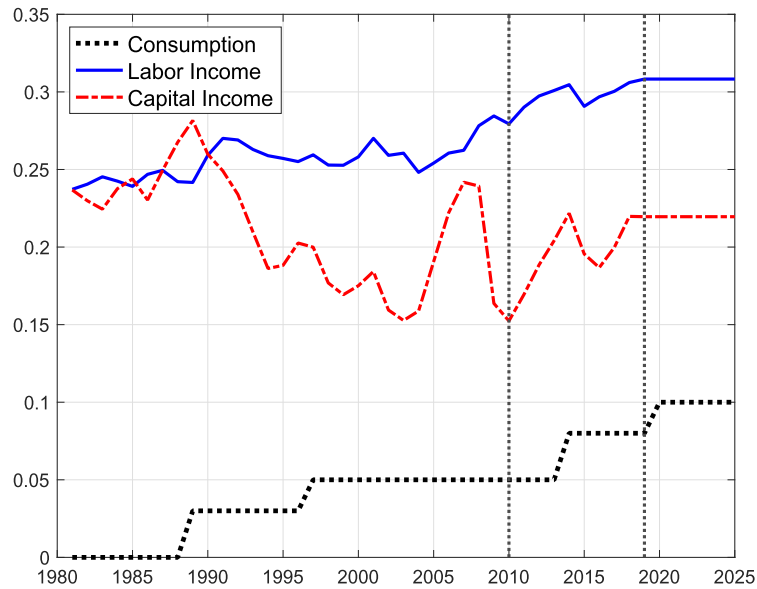
Fig. 6. Tax rates.

Transfer payments, TR_t, include social benefits other than those in kind (such as health care), which are included in G_t. These are mostly public pensions, plus other current net transfers minus net indirect taxes.

Tax rates: Labor income tax rates in Hansen and İmrohoroğlu (2016) came from the estimates of average marginal labor income tax rates by Gunji and Miyazaki (2011) and tax rates on capital income constructed following the methodology in Hayashi and Prescott (2002). Given that this labor tax rate series ended in 2007, we needed to use a different approach in order to obtain tax rates from 1981 to 2019. Tax rates on capital income were computed using the same methodology.

Instead of taking tax rates on capital and labor income from an independent source, we begin with time series on revenue from corporate income taxes (REV_{τ_k}) and labor income taxes (REV_{τ_h}) published by the Japanese government. According to our model, $REV_{\tau_h} = \tau_h Wh = \tau_h(1-\theta)Y$ and $REV_{\tau_k} = \tau_k(r - \delta K) = \tau_k(\theta - \delta\frac{K}{Y}))Y$. These expressions are used to obtain a times series for τ_h and τ_k.

A consumption tax rate of $\tau_{c,t} = 3\%$ was introduced in Japan in 1989, and it was raised to 5% in 1997 and to 8% in 2014. It was ultimately increased to 10% in 2020. In our calibration, we assume that the consumption tax rate stays constant at 10% beyond 2020.

The tax rate on interest from government bonds, $\tau_{b,t}$, is equal to 20% for all time periods. This tax is imposed on the semiannual interest income from coupon-bearing bonds and is withheld (15% income tax plus 5% local tax) at the time the interest is paid. The tax rate on interest payments to the Bank of Japan is 100 percent.

Fig. 6 shows the tax rates on consumption, labor income, and capital income used in this paper.

Demographic change and government expenditures: As mentioned, Japanese government purchases of goods and services, G_t, include Japanese public health expenditures and TR_t, government transfer payments, include public pensions. These components are expected to continue to rise as a result of population aging.

Hansen and İmrohoroğlu (2016) used estimates of the increase in these expenditures between 2010 and 2050 provided in Fukawa and Sato (2009). These authors estimate an increase of 3 percentage points in the ratio of government purchases to output and a 4 percentage point rise in transfer payments to output from 2010 to 2050. According to Fukawa and Sato (2009), the projected increase in government purchases is nearly entirely due to the expected increase in public long-term care expenditures, driven by the increased longevity of the population. Similarly, the projected increase in transfer payments is driven by expected increases in public pension expenditures.[9] These estimates are very similar to those calculated independently by İmrohoroğlu et al. (2016).[10]

[9] The projections in Fukawa and Sato (2009) are based on a system of about 40 regression equations (in addition to definitional relations and equations describing the evolution of the population in different age groups) which is estimated from Japanese data sources over the sample period 1980-2003. The population projections used are the same as those used in this paper. In addition, they assume a rate of growth of real GDP of about 2%.

[10] İmrohoroğlu et al. (2016) build a micro-data-based large-scale overlapping generations model for Japan and incorporate the Japanese pension rules in detail. Using existing pension law and fiscal parameters and the medium variants of fertility and survival probability projections, they produce future time paths for government purchases and transfer payments.
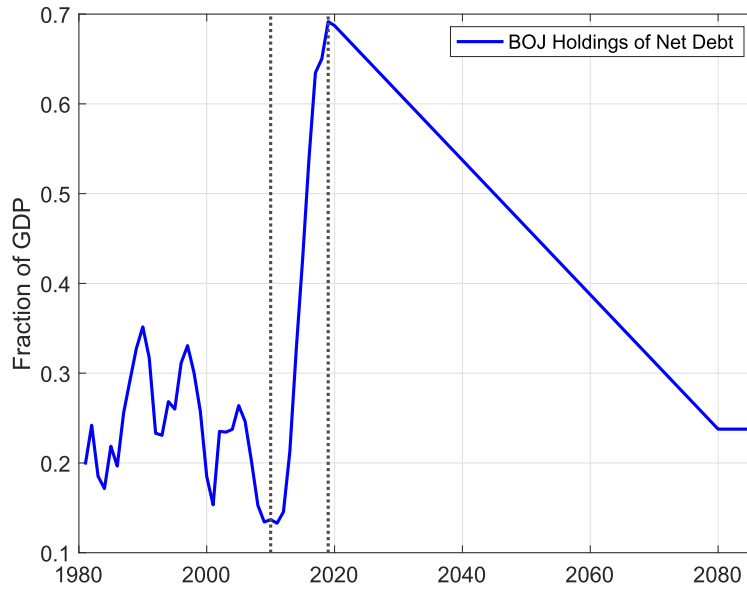
Fig. 7. BOJ holdings of net debt.

Bank of Japan holdings of Japanese government bonds: Values of λ_t from 1981 to 2020 are shown in Fig. 2. While we have no idea what fraction of government debt the Bank of Japan will hold in the future, we assume that this share will linearly decline from 2021 to 2080 and then remain constant at a value equal to the average share from 1981 to 2010.[11] Fig. 7 shows both the actual values of this variable through 2020 and our assumptions about the future.

Our calibration method for assigning values to structural parameters is the same as in Hansen and İmrohoroğlu (2016). As in that paper, we calibrate using data from 1981 to 2010. We do this rather than recalibrating using all available data to make it easier to understand why the predictions for government debt to output made in that paper turned out to be wrong. Holding the calibration constant allows us to focus on the role played by the differences between what actually happened from 2011 to 2019 and our assumptions about government expenditures, revenue, and TFP during that period. Still, while some parameters will be unchanged from our original paper, others need to be recalibrated given that the model employs different tax rates than before.

Technology parameters: Given the data described above, the Cobb-Douglas production function allows us to calculate total factor productivity:

$$A_t = Y_t/(K_t^\theta h_t^{1-\theta}).$$

The capital income share, θ, is set equal to the sample (1981-2010) average of the annual ratio of capital income to our measure of GNP. Similarly, a time series for the depreciation rate of capital is computed following the methodology of Hayashi and Prescott (2002) and $\delta = 0.0842$ is the sample average.

The growth factor of TFP, $\gamma_t = A_{t+1}/A_t$, is computed from the actual data between 1981 and 2019. For 2020 and beyond, we assume that $\gamma_t = 1.015^{1-\theta}$. This implies a growth rate of 1.5% for per capita output along the balanced growth path.

Preference parameters: There are five preference parameters, $\beta, \alpha, \psi, \phi$, and μ, in the utility function given by equation (4). As in our previous work, the parameter ψ (the Frisch elasticity of labor supply) is set equal to 0.5, following Chetty et al. (2012).

For the parameters β, α, and ϕ, we use the sequence of equilibrium conditions given in equations (5), (6), and (7) along with data for exogenous and endogenous variables over the sample period. From this we obtain annual values for each parameter that causes the equilibrium conditions to be satisfied each period. The calibrated parameter values are the sample average from 1981 to 2010. Given that the yearly tax rates in these equations are different from our previous work, the values obtained for these three parameters are somewhat different as well.

$$\beta_t = \frac{(1 + \tau_{c,t+1})\gamma_t^{1/(1-\theta)} c_{t+1}}{(1 + \tau_{c,t})c_t \left[1 + (1 - \tau_{k,t+1})\left(\theta \frac{y_{t+1}}{k_{t+1}} - \delta\right)\right]} \tag{5}$$

[11] Later we will check how our results would change in the extremely unlikely case that the BOJ share of bond purchased remained equal to their 2020 value in perpetuity.
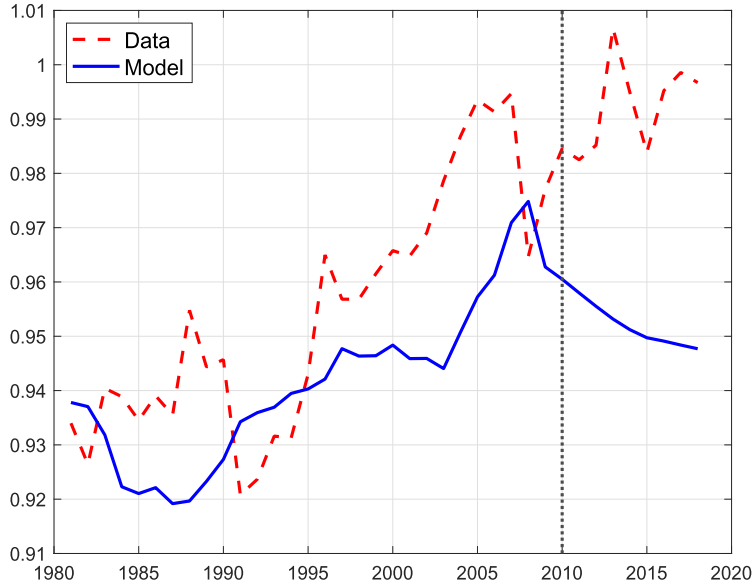
Fig. 8. Real bond prices.

$$\alpha_t = \frac{h_t^{-1/\psi}(1 - \tau_{h,t})(1 - \theta)y_t}{(1 + \tau_{c,t})c_t h_t} \tag{6}$$

$$\phi_t = \eta_t(\mu + b_{t+1}^h)\left[\frac{q_t \gamma_t^{1/(1-\theta)}}{(1 + \tau_{c,t})c_t} - \frac{\beta_t\left[1 - (1 - q_t)\tau_{b,t+1}\right]}{(1 + \tau_{c,t+1})c_{t+1}}\right]. \tag{7}$$

Equation (5) is obtained from the Euler equation for choosing K_{t+1}, equation (6) comes from the static first order condition for choosing h_t, and equation (7) is derived from the Euler equation for B_{t+1}^h. Further details are provided in the appendix.

Note that the equilibrium condition in equation (7) contains the price of government bonds, q_t. In particular, given a time series of η_t and q_t, this equation determines ϕ_t.[12]

The empirical counterpart to q_t we construct is motivated by the fact that government debt in actual economies consists of bond holdings of varying maturities while our model economy includes only one-period discount bonds. In particular, let B_t be the beginning of period debt and P_t be interest payments made in period t, both measured in current Yen. In addition, let F_t be the GNP deflator. We compute the price of bonds in period t as follows:

$$q_t = \frac{B_{t+1}/F_t}{(B_{t+1} + P_{t+1})/F_{t+1}}. \tag{8}$$

Using data on B_{t+1}, F_t, and P_{t+1} over the sample period, we compute an empirical counterpart to q_t and feed these values into the equilibrium conditions to calculate the sample values of the preference parameters.

Fig. 8 shows our empirical time series for government bond prices. Note that bond prices continued to rise (interest rates fell) from 2009 to 2019 while our model predicted that bond prices would fall. This will play an important role in accounting for why debt to output stabilized during this period.

The remaining preference parameter μ, which is the detrended value of μ_t, was chosen to minimize the sum of squared differences between the bond price implied by our model and its data counterpart.

Table 1 summarizes the calibrated values for the structural parameters.

5. Tax revenue and interest payments on debt

In section 2 we looked at how our assumptions about exogenous series including TFP, government purchases and transfer payments behaved from 2011-2019 compared with the assumptions we made based on the work of Fukawa and Sato (2009).

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[12] In equation (7) with ϕ_t constant, there is an inverse relationship between the population growth factor η_t and the price of government bonds q_t, implying a positive relationship between η_t and the interest rate on government bonds. If the population growth rate falls over time, which is a decrease in η_t, the interest rate on bonds is expected to fall. We find that this channel is quantitatively not important. On the other hand, using a Blanchard-Yaari perpetual youth approach to incorporate population aging, Carvalho et al. (2016) and Aksoy et al. (2019) rationalize the observed world-wide decline in real interest rates. Also, Eggertson et al. (2019) use a general equilibrium overlapping generations model to quantify the contributions to the observed decline in interest rates from decreases in mortality and fertility rates.

Table 1
Calibration of structural parameters.

Parameter	Value	
θ	0.3783	Sample Average, 1981–2010
δ	0.0842	Sample Average, 1981–2010
β	0.9502	Equation (5), Sample Average, 1981–2010
α	24.4438	Equation (6), Sample Average, 1981–2010
ψ	0.5	Chetty et al. (2012)
ϕ	0.1273	Equation (7), Sample Average, 1981–2010
μ	1.1	Fit q_t for 1981–2010
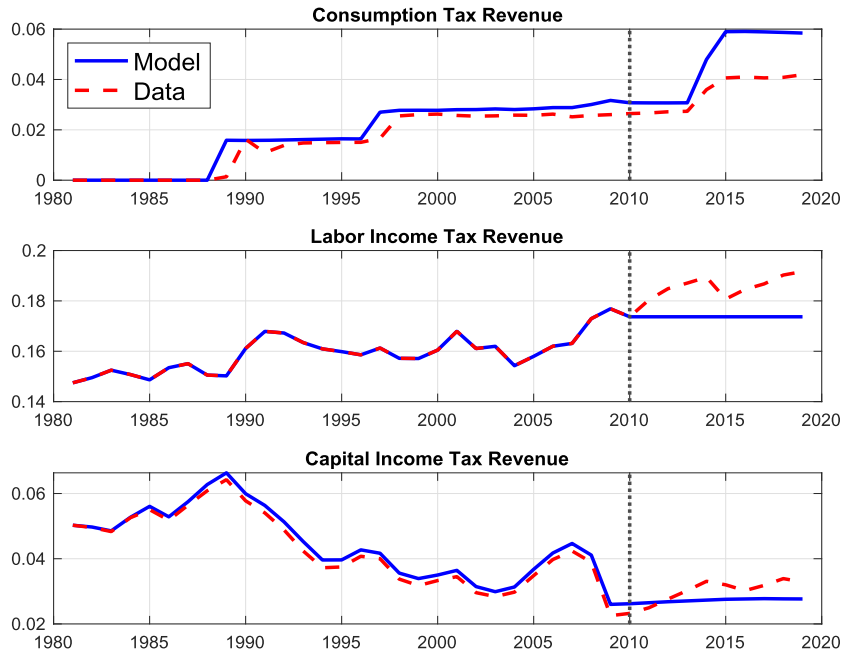
Fig. 9. Tax revenues (fraction of GNP).

In this section, we turn to the endogenous components of the government budget: tax revenue and interest payments on government debt.

Fig. 9 shows tax revenue as a fraction of GNP for each of the primary sources of revenue: the consumption tax, labor income tax, and capital income tax. In each case, data is shown from 1981 to 2019 and model results calibrated through 2010 and our forecasts for revenue from 2011 to 2019. Both labor and capital tax revenue turned out to be higher than was predicted in our previous paper for the 2011-19 period while consumption tax revenue was somewhat less than predicted. Fig. 10 shows that total tax revenue was indeed somewhat larger during this period than we had predicted. The difference, however, is relatively small.

Fig. 11 shows data on net interest payments on government debt to GNP also from 1981 to 2019. The model results use actual data on tax rates, TFP, and government expenditures through 2010 and forecasts from 2011 to 2019. Our model results for the 2011 to 2019 period deviate dramatically from what actually happened. In 2019, interest payments were in fact about 1 percent of GNP while our model predicted they would be about 11 percent. This would appear to be an important factor in why simulated Japanese debt rises so quickly in Fig. 1 relative to the actual data.

6. Quantitative experiments

Our results are presented in the form of four experiments. The first, Experiment 0, is designed to correspond to the calibrated model studied in Hansen and İmrohoroğlu (2016) and is calibrated as described in section 4 using data from 1981 to 2010. It differs from the benchmark experiment in our previous paper due to the labor and capital income tax rates assumed. Forecasts described in section 4 are used to extend tax rates, government expenditures, TFP, and population growth rates into the infinite future. All model results presented in Figs. 8–11 are from Experiment 0.

Experiment 1 uses the same calibration as Experiment 0, but substitutes actual values for population growth rates, TFP, government expenditures, and tax rates from 2011 to 2019 in place of the predictions used in Experiment 0. Predictions are, however, used to fill in all years beyond 2019. The population values are taken from forecasts provided by the National
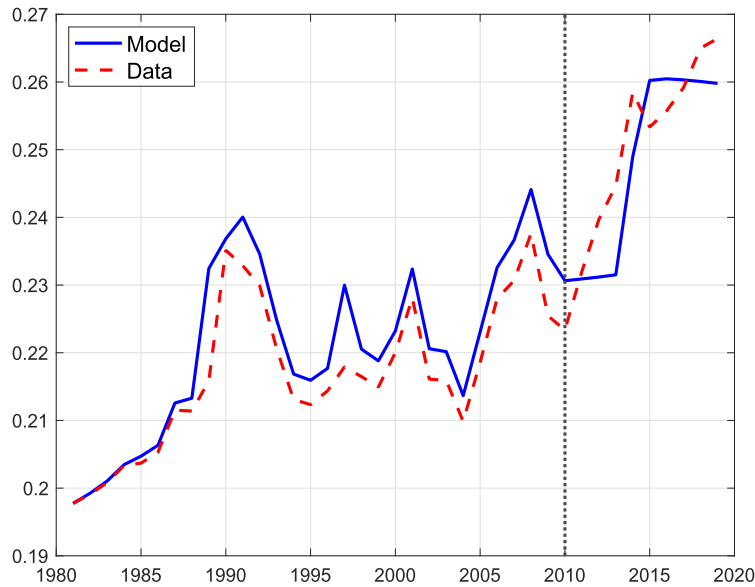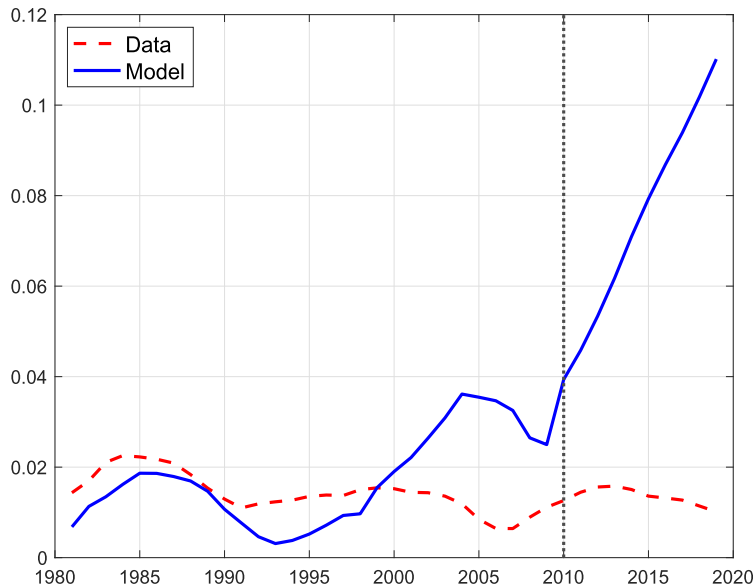
Fig. 10. Total tax revenue (fraction of GNP).

Fig. 11. Net interest payments (fraction of GNP).

Institute of Population and Social Security Research.[13] Beyond 2019 TFP is assumed to grow at a rate $1.015^{(1-\theta)} - 1$ so that output per capita grows 1.5 percent per year. Tax rates for labor and capital income (τ_h and τ_k) are set equal to their 2019 values in 2020 and beyond.[14]

Government expenditures to output (G/Y and TR/Y) are assumed to follow a linear path from 2019 to 2050 so that their values in 2050 are the same as in Experiment 0. The resulting path for $(G + TR)/Y$ is shown in Fig. 12.

Experiment 2 is the same as Experiment 1 except that ϕ is recalibrated to match the low interest rates from 2005 to 2019. That is, ϕ is calibrated to be the average value of ϕ_t computed from equation (7). The resulting value is $\phi = 0.1673$ which reflects an increase in the convenience yield for safety and liquidity associated with government bonds.[15] Del Negro
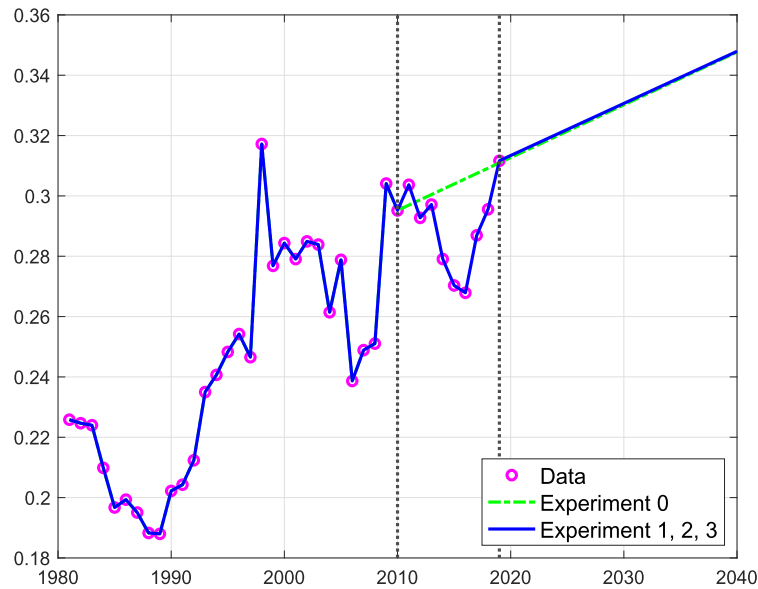

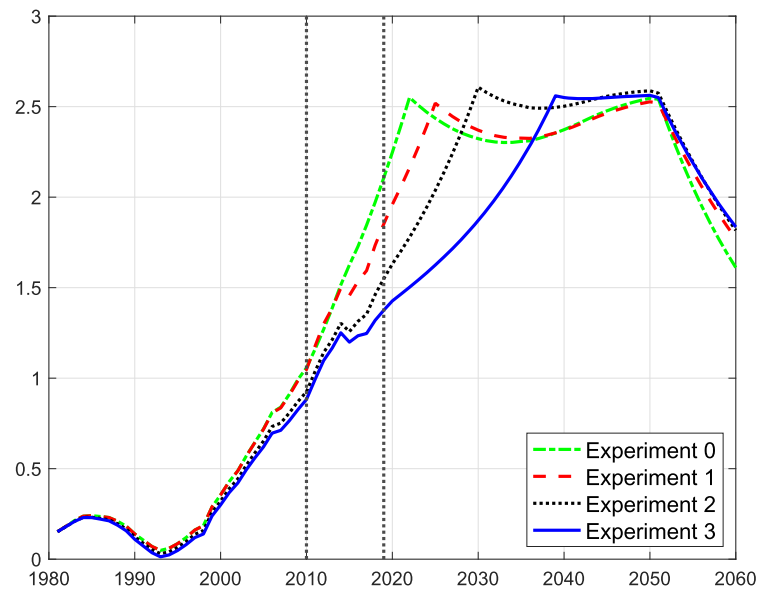**Fig. 12.** (G + TR) to GNP ratios: 1981-2040.

Fig. 13. Debt to GNP ratios: 1981-2060.

et al. (2019) use the increase in the convenience yield for safety and liquidity captured by this preference parameter to account for the significant decline in real interest rates across countries over the past three decades.

In these three experiments, the holdings of government debt by the BOJ are assumed to be zero as in Hansen and İmrohoroğlu (2016). Experiment 3 is the same as Experiment 2 except that the fraction of government debt held by the Bank of Japan is set to the values shown in Fig. 7. The parameter ϕ is set according to the same criteria as in Experiment 2. This implies a value of $\phi = 0.1426$.

Fig. 13 shows government debt to output ratios in each of the four experiments. In Experiment 0, debt to output reaches 250 percent (the point at which our debt sustainability rule is triggered) in 2022.[16] When data on tax rates, government expenditures, TFP, and population growth is extended to 2019 (Experiment 1), the trigger date is delayed three years to 2025. These three extra years are the result of the Japanese government spending less and collecting more revenue than

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[16] In the benchmark case in Hansen and İmrohoroğlu (2016) the rule is triggered in 2021. The only difference between that case and the one here is that different tax rates for labor and capital income are assumed.
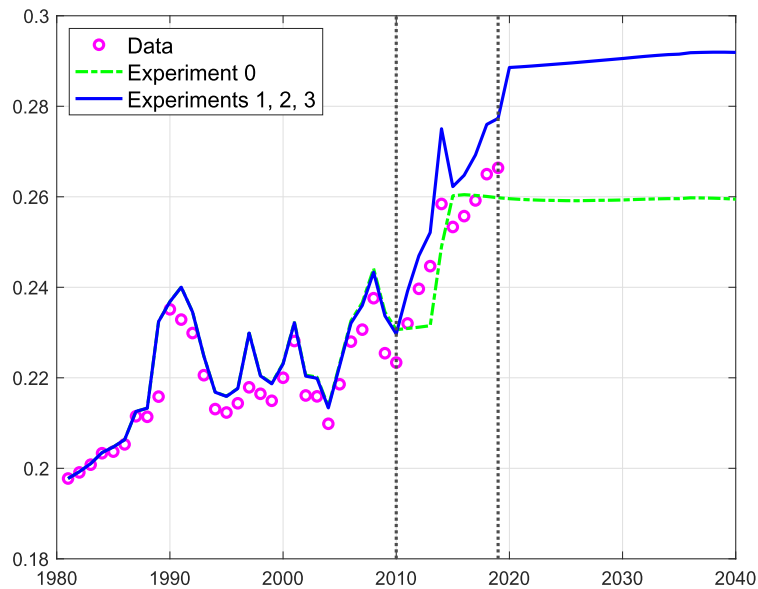
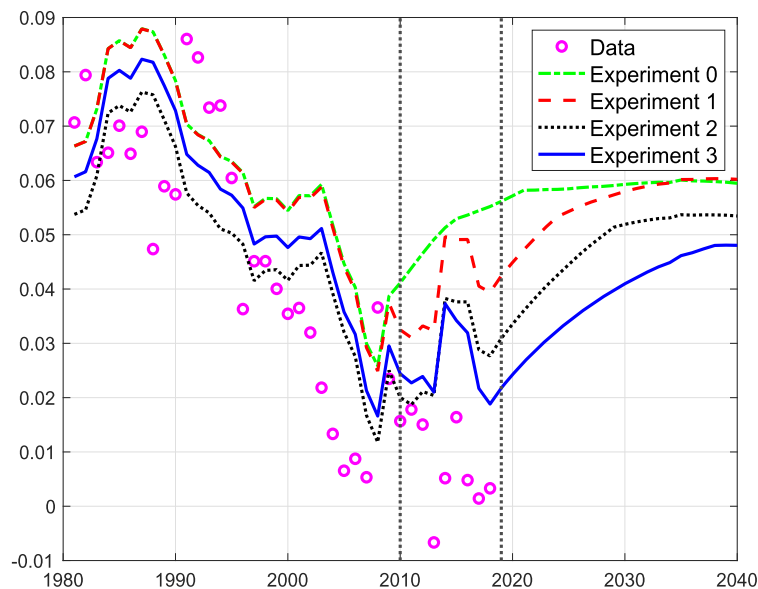**Fig. 14.** Tax revenue to GNP ratios: 1981-2040.

Fig. 15. Interest rates on JGBs: 1981-2060.

assumed in Experiment 0 (or Hansen and İmrohoroğlu (2016)). The lower spending in Experiment 1 is shown in Fig. 12 and the increased revenue in Fig. 14.

Experiment 2 is an attempt to improve on that by calibrating ϕ to raise bond prices (lower interest rates) and hence reduce interest payments on government debt. This change delays the trigger to 2030. That is, in addition to lower spending and higher revenue during the 2011 to 2019 period, the Japanese government benefited from lower interest rates than in the equilibrium of Experiments 0 or 1. Fig. 15 shows how the interest rate on Japanese government bonds ($1/q - 1$) differs across the experiments.

The last experiment (Experiment 3) lowers interest payments further by taking into account BOJ holdings of Japanese government bonds. In particular, the interest earnings from a substantial fraction (λ_t) of recent bonds are returned by the BOJ to the government. In this case, the trigger is delayed to 2039.

Fig. 16 shows ratios of interest payments on government debt to GNP for each of the cases studied. In each case, interest payments begin to soar at different points as debt accumulates. Experiment 3 is interesting in that interest payments to output are barely over 5 percent at the point the trigger is hit in 2039. This is the point at which debt to output first
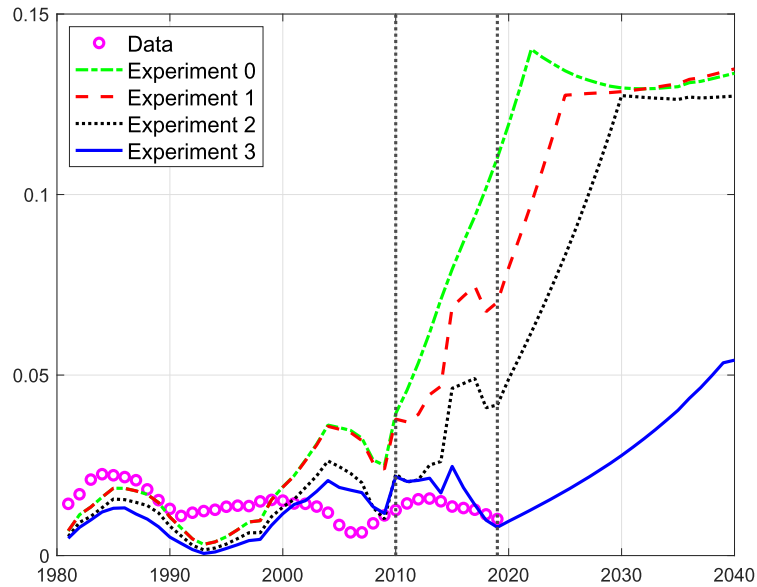
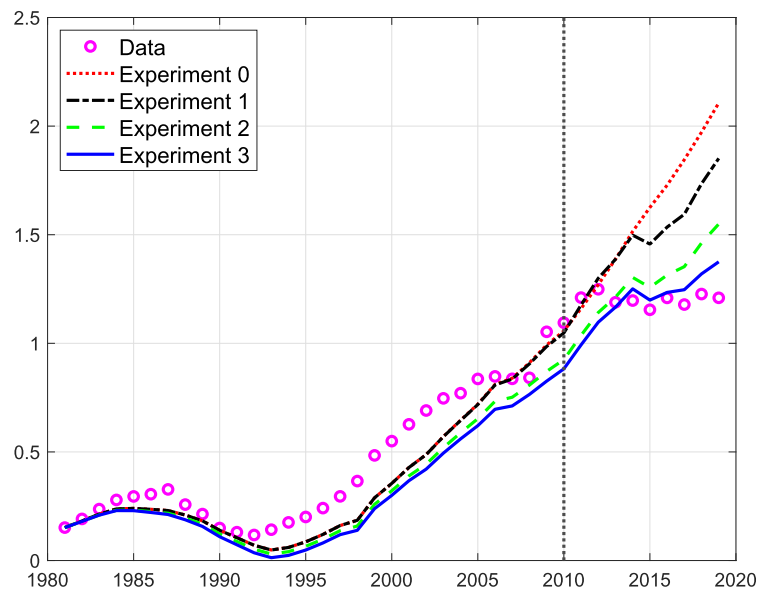**Fig. 16.** Interest payments to GNP ratios: 1981-2040.

Fig. 17. Debt to GNP ratios: 1981-2019.

exceeds 250 percent. In the other cases, this ratio is above 12 percent when the trigger is reached. In addition, spending to output is approaching 35 percent when the trigger is reached in Exercise 3 but is between 31 and 33 percent in the other cases.

Given that interest payments are much lower in Experiment 3 we conducted a variation on that experiment where, instead of λ_t falling after 2020 as in Fig. 7, we held λ constant at the 2020 level into the infinite future. This change only delays the year the trigger is hit by one year, 2040. Hence, while low interest rates and low interest payments incurred by the Japanese government do delay the date when debt to GNP reaches 250 percent, it always reaches that mark before spending to GNP is assumed to level off in 2050. That is, in all cases considered, without an increase in revenue, Japan cannot expect to finance the aging-related expenses predicted by Fukawa and Sato (2009) without net debt to GNP shooting well above historic values.

7. Conclusions

In Hansen and İmrohoroğlu (2016), we computed the increase in tax rates that would be needed to stabilize debt in Japan given projected increases in spending. While we have not repeated that exercise here, increasing spending due to population aging along with constant tax rates into the future is not a fiscal policy that will lead to a stable debt-to-output ratio for Japan according to our model. However, since 2010 Japan has been successful in stabilizing debt with higher taxes, low interest rates, and a cooperative Bank of Japan (see Fig. 17). However, our findings indicate that this success is only temporary. Without increased revenue, Japanese government debt will resume growing to unprecedented levels.

One issue that needs further study is the fact that the net debt to output ratio stabilizes about three years earlier than our model predicts (see Fig. 17). Given that government revenue, spending and BOJ bond purchases closely correspond to actual data, the component of the government budget that accounts for this discrepancy must be interest payments on government debt. Fig. 15 shows that the interest rates determined by our general equilibrium model differ greatly from the observed interest rates in 2013 to 2019. Hence, interest payments in our experiments, shown in Fig. 16, are higher than in the data during much of this period.

Perhaps a more detailed modeling of the balance sheet of the Bank of Japan and a monetary policy targeting near-zero interest rates on long term government bonds would help resolve this discrepancy. This is left for future research.

Data availability

I have already submitted data/code used in the paper.

Appendix A

A.1. Calibration details

Data sources and construction   The main source of data used in this paper comes from an updated version of the rbc.xls file on Professor Fumio Hayashi's web site. In turn, Prof. Hayashi relied mostly on the Japanese National Accounts (compiled by the Economic and Social Research Institute (ESRI), Cabinet Office of the Japanese Government). In particular, we use the Annual Report on National Accounts for 2014 using the 1993SNA (National Accounts of Japan) from 1994 through 2014 is the main data set. We extend this set backward to 1981 using the Annual Report on National Accounts for 2009 (using the 1993SNA also from 1980 to 2009). We start from the level of variables in the former data set and the growth factor in the latter data set to implement this computation.

In a similar fashion, we start from the 1993SNA main data set and extend this forward to 2019 using the Annual Report on National Accounts for 2021 using the 2008SNA. Again, we use the growth factors in the variables in the 2008SNA data set and the level of the data in 2014 in the 1993SNA to complete our data set.[17]

The reason for using the older, 1993SNA is to maintain comparability with our original paper in 2016. We have also constructed an alternative data set starting from the latest 2008SNA and going back to 1981 using the levels in the 2008SNA and the growth factors of the older SNA variables. For our purposes, there is little difference in using this set versus the one described above. For example, our model's Gross National Product variable under both cases are depicted in Fig. 18.

Data adjustments   As Hayashi and Prescott (2000) mention in their appendix, the first adjustment to data is to combine the capital consumption in the Income and Outlays report (historical basis) with that in the balance sheets (replacement cost basis). Please see Hayashi and Prescott (2000) for details or the Excel file updatedrbc.xlsx.

The second adjustment is to subtract the capital consumption for government capital from measured output because government investment in our model is entirely expensed.

Mapping model variables to adjusted national accounts   Consumption, C, in our model is the usual Personal Consumption Expenditures. Government purchases, G, is the sum of government purchases (which includes transfers in kind such as public health expenditures) and government investment. Gross private fixed investment, I, is the sum of private fixed investment and the current account, following the steps in Hayashi and Prescott (2000). Our measure of output, Y, is then $C + I + G$ which is GNP.

Our measure of capital stock, K, excludes government capital but includes capital owned by Japanese in foreign countries, using a method similar to that of constructing the investment series.

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[17] There is also an Annual Report on National Accounts for 1990 using the 1968SNA from 1955 to 1990. There are differences in definitions, changes in asset classes and others from one SNA to another. The 2008SNA is particularly different because of the treatment of the intellectual property products as investment. In addition, the national accounts produce both Income and Outlays and balance sheets and the treatment of capital consumption is not always consistent across the two.
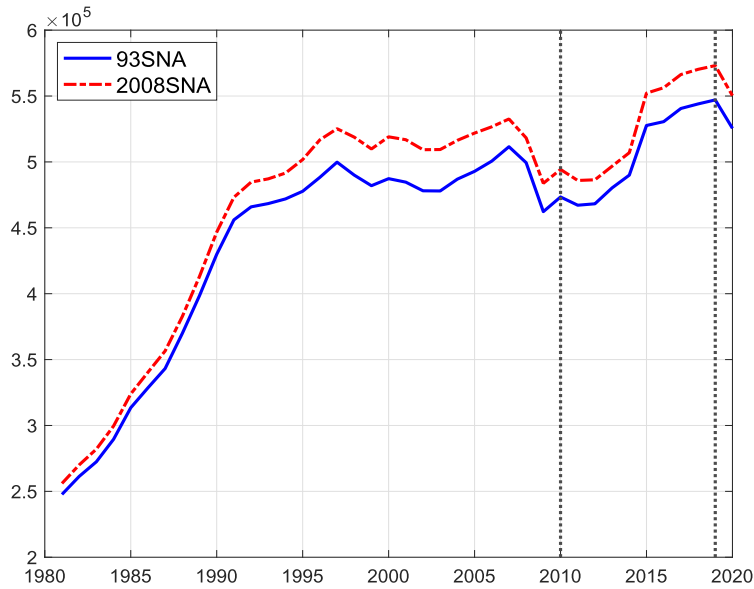
Fig. 18. GNP constructred from 93SNA and 2008SNA.

A.2. Detrended equilibrium conditions

In this subsection we derive the detrended equilibrium conditions to use in solving the model numerically. Given a trending per capita variable Z_t we obtain its detrended per capita counterpart by

$$z_t = \frac{Z_t}{A_t^{1/(1-\theta)}}.$$

The first set of detrended equilibrium conditions is given below.

$$\frac{(1 + \tau_{c,t+1})\gamma_t^{1/(1-\theta)}c_{t+1}}{(1 + \tau_{c,t})c_t} = \beta[1 + (1 - \tau_{k,t+1})(r_{t+1} - \delta)], \tag{9}$$

$$\frac{\phi}{\mu + b_{t+1}^h} + \frac{\beta\eta_t[1 - (1 - q_t)\tau_{b,t+1}]}{(1 + \tau_{c,t+1})c_{t+1}} = \frac{q_t\eta_t\gamma_t^{1/(1-\theta)}}{(1 + \tau_{c,t})c_t}, \tag{10}$$

$$\alpha h_t^{1/\psi} = \frac{(1 - \tau_{h,t})w_t}{(1 + \tau_{c,t})c_t}, \tag{11}$$

$$y_t = k_t^\theta h_t^{1-\theta}, \tag{12}$$

$$\eta_t\gamma_t^{1/(1-\theta)}k_{t+1} = (1 - \delta)k_t + x_t. \tag{13}$$

Equation (9) is the typical Euler equation arising from the choice of capital stock at time t. The bond Euler equation is given by (10). The first order condition for hours worked is shown in equation (11). The production function and the law of motion for capital are given in equations (12) and (13), respectively. The budget constraint for the household is given below in equation (14)

$$(1 + \tau_{c,t})c_t + \eta_t\gamma_t^{1/(1-\theta)}k_{t+1} + q_t\eta_t\gamma_t^{1/(1-\theta)}b_{t+1}$$
$$= (1 - \tau_{h,t})w_th_t + [1 - (1 - q_{t-1})\tau_{b,t}]b_t^h + tr_t - d_t + [1 + (1 - \tau_{k,t})(r_t - \delta)]k_t. \tag{14}$$

The government budget equation is given by equation (15)

$$g_t + tr_t + b_t = q_t\eta_t\gamma_t^{1/(1-\theta)}b_{t+1}^h + \tau_{c,t}c_t + \tau_{h,t}w_th_t$$
$$+ \tau_{k,t}(r_t - \delta)k_t + [\tau_{b,t}(1 - \lambda_t) + \lambda_t](1 - q_{t-1})b_t + d_t. \tag{15}$$

Equation (16) is the detrended debt sustainability rule

$$d_t = \begin{cases} \kappa(b_t - \overline{b}_t \, \overline{y}) & \text{if } b_s/y_s \geq b_{\max} \text{ for some } s \leq t, \\ 0 & \text{otherwise,} \end{cases} \tag{16}$$

where $\overline{y}$ is the value of y_t along the balanced growth path and $\overline{b}$ is the targeted debt to output ratio along the balanced growth path.

Finally, the market clearing conditions are given below in equations (17), (18) and (19).

$$r_t = \theta k_t^{\theta-1} h_t^{1-\theta}, \tag{17}$$

$$w_t = (1-\theta) k_t^{\theta} h_t^{-\theta}, \tag{18}$$

$$c_t + x_t + g_t = y_t. \tag{19}$$

Hence we have 10 equations, (9) through (18), in 10 unknowns $\{c_t, x_t, h_t, y_t, k_{t+1}, b_{t+1}, d_t, q_t, w_t, r_t\}$ at each time period t.

References

Aksoy, Y., Basso, H.S., Smith, R.P., Grasl, T., 2019. Demographic structure and macroeconomic trends. American Economic Journal: Macroeconomics 11 (1), 193–222.

Bamba, M., Weinstein, D.E., 2021. The crisis that wasn't: how Japan has avoided a bond market panic. In: Lipscy, T.H.P. (Ed.), The Political Economy of the Abe Government and Abenomics Reforms. Cambridge University Press, Cambridge, Massachusetts, pp. 239–268.

Blanchard, O.J., 1990. Suggestions for a new set of fiscal indicators. OECD Economics Department Working Papers, No. 79.

Braun, R.A., Joines, D.H., 2015. The implications of a graying Japan for government policy. Journal of Economic Dynamics and Control 57, 1–23.

Broda, C., Weinstein, D.E., 2005. Happy news from the dismal science: reassessing Japanese fiscal policy and sustainability. In: Ito, H.P.T., Weinstein, D.E. (Eds.), Reviving Japan's Economy. MIT Press, Cambridge, Massachusetts, pp. 40–78.

Carvalho, C., Ferrero, A., Nechio, F., 2016. Demographics and real interest rates: inspecting the mechanism. American Economic Review 88, 208–226.

Chen, K., İmrohoroğlu, A., İmrohoroğlu, S., 2006. The Japanese saving rate. American Economic Review 96 (5), 1850–1858.

Chetty, R., Guren, A., Manoli, D., Broda, A.W., 2012. Does indivisible labor explain the difference between micro and macro elasticities? A meta-analysis of extensive margin elasticities. In: Acemoglu, D., Parker, J., Woodford, M. (Eds.), NBER Macroeconomics Annual. The University of Chicago Press, Massachusetts, pp. 1–56.

Cooley, T.F., Prescott, E.C., 1995. Economic growth and business cycles. In: Cooley, T.F. (Ed.), Frontiers of Business Cycle Research. Princeton University Press, Princeton, New Jersey, pp. 1–38.

Del Negro, M., Giannone, D., Giannoni, M.P., Tambalotti, A., 2019. Global trends in interest rates. Journal of International Economics 118, 248–262.

Doi, T., 2008. Simulation analysis of policies to ensure Japan's fiscal sustainability. Mita Journal of Economics 100 (4), 131–160.

Doi, T., Hoshi, T., Okimoto, T., 2011. Japanese government debt and sustainability of fiscal policy. Journal of the Japanese and International Economies 25 (4), 414–433.

Eggertson, G.B., Mehrotra, N.R., Robbins, J.A., 2019. A model of secular stagnation. American Economic Journal: Macroeconomics 11 (1), 1–48.

Fukawa, T., Sato, I., 2009. Projection of pension, health and long-term care expenditures in Japan through macro simulation. The Japanese Journal of Social Security Policy 8 (1), 33–42.

Gunji, H., Miyazaki, K., 2011. Estimates of average marginal tax rates on factor incomes in Japan. Journal of the Japanese and International Economies 25 (2), 81–106.

Hansen, G., İmrohoroğlu, S., 2016. Fiscal reform and government debt in Japan: a neoclassical perspective. Review of Economic Dynamics 21, 201–224.

Hayashi, F., Prescott, E.C., 2002. The 1990s in Japan: a lost decade. Review of Economic Dynamics 5 (1), 206–235.

İmrohoroğlu, S., Kitao, S., Yamada, T., 2016. Achieving fiscal balance in Japan. International Economic Review 57, 117–154.

İmrohoroğlu, S., Sudo, N., 2011. Productivity and fiscal policy in Japan: short-term forecasts from the standard growth model. Monetary and Economic Studies, Institute for Monetary and Economic Studies, Bank of Japan 29, 73–106.

Kitao, S., 2015. Fiscal cost of demographic transition in Japan. Journal of Economic Dynamics and Control 54, 37–58.

Krishnamurthy, A., Vissing-Jorgensen, A., 2012. The aggregate demand for treasury debt. Journal of Political Economy 120 (2), 233–267.